

Stanford Verification Group
Report No. 16

February 1980

Computer Science Department
Report No. STAN-CS-80-789

ADA EXCEPTIONS: Specification and Proof Techniques

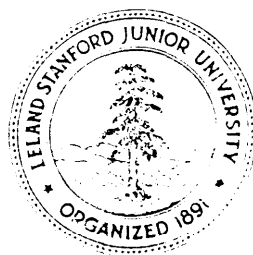
by

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Research sponsored by

Advanced Research Projects Agency
and
Rome Air Development Center

COMPUTER SCIENCE DEPARTMENT
Stanford University



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ABSTRACT

A method of documenting exception propagation and handling in Ada programs is proposed. Exception propagation declarations are introduced as a new component of Ada specifications. This permits documentation of those exceptions that can be propagated by a subprogram. Exception handlers are documented by entry assertions. Axioms and proof rules for Ada exceptions are given. These rules are simple extensions of previous rules for Pascal and define an axiomatic semantics of Ada exceptions. As a result, Ada programs specified according to the method can be analysed by formal proof techniques for consistency with their specifications, even if they employ exception propagation and handling to achieve required results (i.e. non error situations). Example verifications are given.

This research was supported by the Advanced Research Projects Agency of the Department of Defense under Contract MDA 903-80-C-0159 and Rome Air Development Center under Contract F 30602-80-C-0022. The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of Stanford University, or any agency of the U. S. Government.

ADA EXCEPTIONS : SPECIFICATION AND PROOF TECHNIQUES

D. C. Luckham and W. Polak

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This research was supported by the Advanced Research Projects Agency of the Department of Defense under Contract MDA 903 -- 80 -- C -- 0159 and Rome Air Development Center under contract F 30602 -- 80 -- C -- 0022

1. Introduction

Exceptions in the Ada language [I] are intended to provide a facility for dealing with errors and exceptional situations arising during program execution. An exception is an event that causes suspension of normal program execution — called raising an exception. Actions in response to the occurrence of an exception are coded in special units called handlers. When an exception is raised, execution of a handler replaces execution of the current unit. The choice of handlers is dynamic — i.e., determinable only at **runtime**. **Ada** exceptions are clearly a powerful programming facility and a study of their semantics and proof is needed.

The obvious semantics of raising an exception is that of a jump to the corresponding handler. In fact, the conventional go to rule [10, 11] is **sufficient** to describe exceptions with static association of handlers. Problems arise in the case where an exception is not handled locally in a subprogram and the association of the exception with the handler is dynamic. The answer lies in requiring documentation for exceptions that are propagated by procedures or functions; it is then possible to modify the procedure call rule to describe the dynamic association of handlers with exceptions at call time.

We propose a method of specifying exceptions in Ada programs. Axioms and proof rules based on these specifications are given. These define an axiomatic semantics of Ada exceptions, and make possible the verification (proof of consistency with specifications) of Ada programs containing exceptions. Our specifications and proof rules apply to programs with exceptions irrespective of whether exceptions are used only for error situations or as a method of programming normal program behavior (e.g., the program is intended to continue running when exceptions are raised and handled). The proof rules are simple extensions of rules of conventional **axiomatics** e.g. those of Pascal [7]. Our results show that if the programmer follows a simple specification discipline, then the use of exceptions as a normal programming tool does not greatly complicate the analysis of programs for consistency with specifications.

We restrict presentation of our method to simple cases. In particular, the semantics assumes call by reference implementation for out and **in** out parameters. Similar proof rules can be given for other implementations of parameter passing. Extensions to more complicated programs, e.g. Ada packages or Ada programs in which exceptions are propagated beyond their scope, are outlined in section 5 and the appendix. We will not deal with the task failure exception propagated between tasks or with suppression of exceptions.

Throughout this paper we will use the notation and terminology of the Ada report [1], except in logical assertions, where we use $a[i]$ to denote the i -th element of array a to improve readability. The word subprogram will be used for a

function or procedure.

A statement of the form **assert** <assertion> is not the assert statement of Ada; <assertion> is a logical assertion and is not intended to be executed. Logical assertions are more general specification statements than Ada conditions, but they obey the normal scope rules: all free variables **are visible program variables**.

2. Specifying Exceptions

The exception facility of Ada includes

- declarations of user **defined** exceptions
- a raise statement that causes an exception, and
- the definition of exception handlers, i.e. code that is to be **executed** whenever a particular exception is raised.

We now **define** the **formal** documentation that has to be provided for **exceptions**.

(1) No documentation is required for exception declarations and raise statements.

(3) For each exception handler the programmer will be required to provide an assertion that is to be true whenever one **of the exceptions in** the when clause is raised. Following Ada we will use the syntax:

```
exception-handler ::=
    when exception { | exception }
    assert assertion => sequence_of_statements
```

(3) An exception that is raised in a subprogram and is not handled locally is said to be propagated by this subprogram ([1] 11.34). We introduce an exception propagation declaration and require that each exception propagated by a **subprogram** be specified in the subprogram's propagation declaration. Analogous to a handler, a propagation declaration has to provide an assertion that is to be true whenever one of the exceptions in the propagate clause is propagated.

Propagation declarations are placed at the end of the Ada subprogram specification ([1] 6.2). We consider propagation declarations as part of a (to be defined) specification language for Ada. As such, they are of the same nature as global, entry, and exit specifications **for** procedures and functions **of** Pascal Plus [11]. We use the **syntax**:

```
propagation_declaration ::= =
    propagate exception { | exception }
    assert assertion
```

subject to the normal scope rules. Note, that a call to a procedure that propagates an exception is a statement that “**can** raise an exception”. We restrict such calls to the scope of the exception; the use of propagate declarations **now makes this** restriction compiler checkable. In section 5 we show how these specification requirements may be relaxed by use of the *others* clause. The appendix contains an example of how our specifications will require **modifications** of an Ada program that propagates an exception out of the scope of the declaration of that exception.

In a scope *S* where an exception *E* can be raised either by a raise statement or a subprogram call the assertion **associated** with *E* in this scope is defined as follows:

- If *S* is a block with a handler for *E* then the **assertion** of this handler is the associated assertion,
- *If* *S* is a block without a handler for *E* then the assertion associated with *E* in *S* is the assertion associated with *E* in the scope immediately enclosing *S*.
- If *S* is a subprogram, then the assertion of the propagation declaration for *E* is associated with *E*.

Remark: An exception has an associated **assertion** in every scope where **it can be** raised.

For example, consider the following program.

```

procedure  $p(\dots)$ 
  propagate  $E$  assert  $A$  is
begin
  begin
    ...
    raise  $E$ ;      —  $B$  is associated assertion
    ...
  exception
    when  $E$  assert  $B \Rightarrow \dots$ 
  end;
  begin
    ...
    raise  $E$ ;      —  $A$  is associated assertion
    ...
  end
end  $p$ 

```

3. Axiomatic Semantics of Exceptions

3.1. Raising Exceptions

Wherever the statement **raise E_i** occurs, the above specification principles guarantee that there is an assertion A_i associated with E_i at that point. The semantics of **raise E_i** is described by the **raise axiom**:

$$\{A_i\} \text{ raise } E_i \{false\}.$$

3.2. Blocks

The rule of exceptions given below requires that any exception handler achieves the exit assertion **of** the block in which it is contained. The handler may assume the truth *of* the assertion in the when clause.

Let C be the block

```

begin
     $S_0$ 
exception
    when  $E_1$  assert  $A_1 \Rightarrow S_1$ 
    when . . .
    ...
    when  $E_n$  assert  $A_n \Rightarrow S_n$ 
end

```

then we have the following rule *of* exceptions:

$$\frac{\{P\} S_0 \{Q\}, \{A_1\} S_1 \{Q\} \dots \{A_n\} S_n \{Q\}}{\{P\} C \{Q\}}$$

3.3. Procedure calls

Rules given in 3.1 and 3.2 are **sufficient** to describe the effect of exceptions handled in the subprogram in which they are raised. We now propose a modified procedure call rule that describes the effect **of** an exception propagated by a procedure. We will restrict our attention to procedures only; a similar treatment for functions is immediate.

Let p be a procedure that propagates the exceptions $E_1 \dots E_n$ and let $A_1 \dots A_n$ be the corresponding assertions in the propagate declaration of p . Let B_j be the assertion associated with E_j in the calling environment according to the principles in 2. The procedure call rule will require that $A_j \rightarrow B_j$ for all exceptions E_j propagated by p with proper substitutions for in parameters and quantification for out and in-out parameters.

The procedure call rule presented below is taken from [9], adapted for **Ada** and exception handling. Let p be a procedure with f_i, f_o , and f_{io} being the formal in, out, and in out parameters of p respectively. We assume that the correctness of the body of p has been established with respect to the input condition $I(f_i, f_{io})$ and output condition $O(f_i, f_o, f_{io})$. Let a_i, a_o , and a_{io} be the corresponding actual parameters of a call to p , then this call is described by the rule:

$$\frac{\begin{array}{l} P \rightarrow I(a_i, a_{io}) \wedge \forall a_o, a_{io}. (O(a_i, a_o, a_{io}) \rightarrow Q) \\ P \rightarrow I(a_i, a_{io}) \wedge \forall a_o, a_{io}. (A_1(a_i, a_o, a_{io}) \rightarrow B_1) \\ \dots \\ P \rightarrow I(a_i, a_{io}) \wedge \forall a_o, a_{io}. (A_n(a_i, a_o, a_{io}) \rightarrow B_n) \end{array}}{\{P\} p(a_i, a_o, a_{io}) \{Q\}}$$

where the clause $P \rightarrow I(a_i, a_{io}) \wedge \forall a_o, a_{io}. (A_j(a_i, a_o, a_{io}) \rightarrow B_j)$ has to be proven for each propagated exception E_j with propagate assertion A_j and B_j associated with E_j in the calling environment. Note that the treatment of exceptions is similar to that of the normal output condition; raising exceptions may be viewed as just another exit of p .

The above rule assumes p to be a restricted procedure without any global variables. It may be extended to provide for global variables. Furthermore, we assume an implementation of procedure calls using call by **reference** for out and in out parameters. Our rules can be adapted for other procedure call mechanisms. Also, other procedure call rules (e.g. [2,5,7]) can be extended to include the conditions $A_i \rightarrow B_i$.

4. Examples

4.1.

Consider the procedure **search** with in parameters a, n , and x and out parameter i with the entry assertion **true** and the exit assertions $1 \leq i \leq n \wedge x = a[i]$. In case that the value x is not in a , **search** will raise the exception **notfound**. For this exception **search** contains a propagation with the assertion $\forall j. (1 \leq j \leq n \rightarrow x \neq a[j])$. Altogether we have the procedure declaration

type **narray** is array (1..n) of integer;

procedure **search**(a : in **narray**; n : in integer; x : in integer; i : out integer)

entry **true**

exit $1 \leq i \leq n \wedge x = a[i]$

propagate not found assert $\forall j. (1 \leq j \leq n \rightarrow x \neq a[j])$ is

begin

...

raise **notfound**;

...

end

A proof of the body of **search** may use the axiom

$$\{\forall j. (1 \leq j \leq n \rightarrow x \neq a[j])\} \text{ raise notfound } \{false\}$$

Procedure **search** could be used in the following context (m, y , and b are declared in some global scope):

```

assert true;
begin
  search(b, m, y, k);
exception
  when notfound assert  $\forall j.(1 \leq j \leq m \rightarrow y \neq b[j]) \Rightarrow$ 
    k = 0
end;
assert {y = b[k]  $\vee$  ( $\forall j.(1 \leq j \leq m \rightarrow y \neq b[j]) \wedge k = 0$ )}

```

Consistency of this block with its specifications can be shown by

$$\forall k \left((1 \leq k \leq m \wedge y = b[k]) \rightarrow \right. \\ \left. y = b[k] \vee (\forall j.(1 \leq j \leq m \rightarrow y \neq b[j]) \wedge k = 0) \right) \quad (1)$$

$$\forall k (\forall j.(1 \leq j \leq m \rightarrow y \neq b[j]) \rightarrow \forall j.(1 \leq j \leq m \rightarrow y \neq b[j])) \quad (2)$$

$$\forall j.(1 \leq j \leq m \rightarrow y \neq b[j]) \rightarrow \\ y = b[0] \vee (\forall j.(1 \leq j \leq m \rightarrow y \neq b[j]) \wedge 0 = 0) \quad (3)$$

Formula (1) proves that the exit assertion of the call to *search* implies the exit assertion of the block (note, that *P* and *I* of the procedure call rule are both **true**). This proves consistency if the call to search terminates normally.

Formula (2) proves that the assertion associated with *notfound* inside search implies the assertion associated with *notfound* in the calling environment (see call rule). This proves that *notfound* is propagated by the call to search only in situations that the handler “**expects**”.

Finally, (3) proves that the handler does indeed establish **the exit condition** of the block (i.e. the handler “**handles**” correctly).

4.2.

The methods presented in this paper are also useful in proving **that an exception** will never be raised. If we have an exception associated with the assertion **false**, then the corresponding handler will **never** be executed since its precondition is never satisfied. Consider the example:

```

assert  $\exists j.1 \leq j \leq m \wedge y = b[j]$ ;
begin
  search(b, m, y, k);
exception
  when not found arrert false => < empty >
end;
arrert {y = b[k]}

```

The corresponding verification conditions are (numbering **as** in previous example):

$$\begin{aligned}
& \exists j.(1 \leq j \leq m \wedge y = b[j]) \rightarrow \\
& \quad \forall k.((1 \leq k \leq m \wedge y = b[k]) \rightarrow y = b[k]) \tag{1} \\
& \exists j.(1 \leq j \leq m \wedge y = b[j]) \rightarrow \forall k.(\forall j.(1 \leq j \leq m \rightarrow y \neq b[j]) \rightarrow \text{false}) \tag{2} \\
& \text{false} \rightarrow y = b[k] \tag{3}
\end{aligned}$$

5. Extensions

5.1. Use of the “others” construct

Our method also applies if an exception handler contains the **otherr** clause. If a block contains such a handler,

```

exception
  ...
  when otherr arrert A => ...

```

then in this block *A* is the assertion associated with all exceptions that are raised in the block and are not contained in another when clause in the handler. All proof rules and the raise axiom apply unchanged.

The situation is different in propagate declarations. If we allow an **others** clause in propagate declaration⁸ then any exception that is not explicitly mentioned in this propagate assertion can potentially be propagated via the **otherr** clause. Our method can be extended to handle this case if we accept a more complicated procedure call rule.

The general principle of the call rule remains the same: for each exception that can potentially be propagated by the called procedure we have to prove that the assertion **specified for** this exception in the propagate declaration implies the assertion associated with the exception in the calling environment.

First, note that a scope containing a call to a procedure with an **otherr** clause in the propagation declaration must define an associated assertion for every exception that may be raised in it; thus in general **its** handler or propagate declaration also **has to have an otherr** clause. Given **the** above situation we can classify all exceptions as follows.

S_o — set of exceptions explicitly named in handlers of propagate declarations for calling environment.

S_i — set of exceptions explicitly named in propagate declaration for called procedure.

B_j — associated assertion for each $E_j \in S_o$.

A_j — assertion specified in propagate declaration for each $E_j \in S_i$.

B_∞ — assertion for **others** clause in handler (or propagate declaration) of calling environment.

A_∞ — assertion for **others** clause in propagate declaration of called procedure.

To prove that a procedure call is correct we have to verify that:

- a.) $A_j \rightarrow B_j$ for all $E_j \in S_i \cap S_o$,
- b.) $A_j \rightarrow B_\infty$ for all $E_j \in S_i - S_o$,
- c.) $A, \rightarrow B_j$ for all $E_j \in S_o - S_i$, and
- d.) $A, \rightarrow B_\infty$.

The procedure call rule has to be augmented by these **cases, where** proper substitutions have to be performed as described in section 3.

A better proposal to reduce the number of necessary propagate declarations is to introduce a single global exception, error. If a **user** does not want to include each propagated exception with a corresponding **assertion in a propagate declaration** he may include the handler

exception
when **others assert A ==> raise** error;

in any subprogram and provide a propagate declaration for error. This **conversion** has been discussed in 12.5. of the **Ada rational but was rejected on efficiency considerations.**

5.2. Application to Packager

The method described in this paper is **also** applicable to exceptions propagated **from** within modules (or Ada packages). It applies directly to calls of module procedure8 but requires some changes when applied to module bodies. Let us

briefly outline the key idea without going into the detail⁸ of a particular method of specification and **verification** of modules,

Let us assume that to the using program a module represents an abstract object together with certain operations (module procedures). Some of these may propagate exceptions, **overflow** in a stack for instance. Part of the visible specification of module procedures are exception propagation declaration⁸ using abstract specifications. Then the procedure **call** rule of section 3.3 applies to **module procedure** calls as well.

For example, the package in [1] 12.3, could be **specified as**

```
generic (size: integer; type elem)
package stack is
  overflow, underflow: exception;
  procedure push(E: in elem)
    propagate overflow assert full(stack);
  procedure pop(E: out elem)
    propagate underflow assert empty(stack);
end stack
```

This additional documentation is valuable to the programmer even if a formal **verification** is not attempted. The Ada syntax merely states that any of the module procedures may raise any one of the listed exceptions. The propagate declarations give the extra information that, for example, **pop will never raise the exception overflow**.

The extra problem corner in **verifying** that exceptions are raised correctly in the stack body. Internal assertions associated with exceptions may refer to local variables of the module body in addition to parameters of procedures. Thus the condition **full(stack)** may internally correspond to a condition **length = size** for some variable **length** local to the module body. Consequently, **for overflow the following raise axiom is valid within the stack body:**

{length = size} raise overflow {false}.

A “module declaration rule” has to **enforce** that the visible propagate assertion for an exception is equivalent to the assertion in the body that describes the internal state in which the exception is propagated.

5.3. Predefined exceptions

It is conceivable that predefined exceptions can be handled with our method. The **situation** here is complicated by the fact that predefined **exceptions are**

usually raised implicitly by numerous different statements and expressions. Therefore, all proof rules have to be changed to account for exceptions. For example, the assignment $x \leftarrow y/z$ in a scope where \mathbf{A} is the assertion associated with *divide_error* is not described by the standard assignment axiom $\{P\}_{y/z}^x x \leftarrow y/z \{P\}$ but rather by the rule

$$\frac{Q \wedge z \neq 0 \rightarrow P \Big|_{y/z}^x, Q \wedge z = 0 \rightarrow \mathbf{A}}{\{Q\} x \leftarrow y/z \{P\}}$$

This rule differs from the standard assignment axiom in having an extra premiss requiring proof that the associated assertion, \mathbf{A} , is true when divide-error is raised. Similar extra premisses must be added to the proof rules for many other language constructs leading to an exponential increase in the number of subproblems such as $Q \wedge z = 0 \rightarrow \mathbf{A}$. If, however, \mathbf{A} is the assertion *true*, then the additional subproblems are trivial. In many **cases** associating true with an exception handler will be **sufficient** to verify that a program continues correctly after an exception has been raised.

Since predefined exceptions frequently correspond to “runtime errors”, an alternate approach towards predefined exceptions is to verify that for certain sets of input values these **exceptions** will never be raised. Positive results in this direction are reported in [4].

6. Conclusion

We have given an axiomatic semantics for Ada exceptions based on a method of formally specifying exceptions; the method extends to packages in a natural way. The required documentation appears very natural and should not be too demanding for the programmer. The propagate declaration is a helpful specification even in cases where a formal verification is not desired as is shown by the documentation of the stack module in 5.1. The additional information provided by propagate declarations can be used to check **for** certain semantic restrictions at compile time, e.g. to check that no exception is propagated out of its scope, or that all exceptions that may be raised within a subprogram and not propagated are handled within that subprogram.

The question of how the task failure exception can be treated in a multi task program and whether this can be done consistently with our method has to be investigated in future research.

Acknowledgement

We are grateful for helpful conversations with Jean Ichbiah and Steve German.

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Appendix

Propagating exceptionr out of their scope

The Ada rationale [1] gives the following example of exceptions propagated out of their scope (12.5.2):

```
package D is
  procedure A;
  procedure B;
end

procedure Outside is
begin
  ...
  D.A;
  ...
end

package body D is
  error: exception;
  procedure A is
  begin
    ... raise error;
  end;

  procedure B is
  begin
    ...
    Outside;
    ...
  exception
    when error => ...
  end;
end D
```

— **Outside** is declared outside the scope of error

— **D.A** may propagate error in which case **Outside** will propagate “something” that cannot be named syntactically in the declaration of **Outside**

— scope of error-exception

— call to **Outside** may propagate ‘something’ which the Ada implementation will recognize as error

In this example the scope of the exception error is the body of **D**. The call **D.A** in **outside** may propagate error and cause error to be propagated from **Outside**. Since **Outside** is declared outside the scope of error, this exception cannot be named explicitly in a handler **for Outside**. It can be handled only by

an others clause. The programmer has essentially lost the ability to distinguish error from other exceptions and to handle it distinctly. In programs, exceptions propagated out of their scope become anonymous. Interestingly, the Ada implementation will maintain the identity of error when it is propagated outside its scope. Therefore, if it is propagated back into its scope again — **as** in this example when **Outside** is called from **B** — it will be handled by a handler for error and **not by others**. The implementation thus behaves as though all exception propagations **are** within the scope of the exception.

If Ada is extended to require propagate declarations (without an **others** clause), propagation of exceptions beyond their **scope** is syntactically illegal. For example the package **D** must be modified. Since **A** can propagate error and **A** is a visible procedure *of D*, the exception error has to be declared in the visible part of **D**. Then the exception, **D.error**, is visible to all scopes which **can call D.A**. Using our documentation, the above example could be written **as**:

```

package D is
  error: exception;           — D.error exception declaration
  procedure A                 — all calls to D.A are within the scope of
    propagate error assert P; D.error
  procedure B;
end

procedure Outside
  propagate D.error assert Q
is
begin
  ...
  D.A;                       — Q is the assertion associated with D.error
  ...                          — a handler for D.error could also be
  ...                          included in outside
end

...
...

```

If one chooses **to** allow **others** clauses in propagate declarations **as** indicated in section 5, then **propagation** of exceptions out of their scope becomes possible. In this **case our method** will work correctly according to **the Ada semantics**.

Proof of file transfer example

We consider a slightly modified version of the file transfer example given in the Ada rationale in 12.3.3. In addition to propagate and handler assertions, the program is documented with **entry**, **exit**, **initial**, and **invariant** assertions as introduced for Pascal Plus in [11].

We assume that the system routines **get** and **put** have the following specifications:

```
generic(type t)
  procedure get(j: in out file of t; x: out t)
  initial j = f0
  entry true
  exit  $\neg \text{empty}(f0) \wedge f0 = \text{append}(\text{list}(x), j)$ 
  propagate end_of_file assert  $\text{empty}(f0) \wedge j = f0$ ;
```

```
generic(type t)
  procedure put(f: in out file of t; x: in t)
  initial j = f0
  entry true
  exit j =  $\text{append}(f0, \text{list}(x))$ ;
```

We prove the correctness of the following transfer procedure; **get** and **put** are assumed to be appropriate instances of the generic procedures declared above:

```
procedure transfer(in j: in out file of t; out j: out file of t)
  initial in j = in f0
  entry empty(out j)
  exit out j = in f0
is
  c: t;
begin
  invariant  $\text{append}(\text{out } j, \text{in } j) = \text{in } f0$ 
  loop
    get(in f, c);
    put(out f, c)
  end loop;
exception
  when end_of_file assert  $\text{empty}(\text{in } f) \wedge \text{out } j = \text{in } f0 \Rightarrow$ ;
end;
```

The correctness is established by proving the following verification conditions. Note, that instead of introducing universal quantifiers we use new variables to

construct formulas logically equivalent to the quantified formulas in the procedure call rule.

1) **Correctness of the handler:** The handler code (empty statement) has to achieve the exit condition of the block:

$$(empty(in\ j) \wedge out\ j = inf_0 \rightarrow outf = inf_0)$$

This verification condition is trivially true.

2) **Correctness of raising the exception:** The propagate assertion for *end_of_file* in *get* must imply the handler assertion. Since *get* is called within the loop, it may be assumed that the loop invariant is true when a propagation occurs; the verification condition has the form

$$invariant \wedge propagate\ assertion \rightarrow handler\ assertion:$$

inf_0 is a new variable denoting the value of the actual parameter in *j* of the call to *get* when propagation occurs.

$$\begin{aligned} & (append(out\ j, inf) = inf_0 \wedge in\ j = inj_0 \wedge empty(in\ j) \\ & \rightarrow \\ & empty(inf_0) \wedge outf = inf_0) \end{aligned}$$

This verification condition simplifies to true, observing that $append(outj, inf) = outf$ whenever $empty(in\ j)$.

3) **Correctness of entering and leaving the loop:** The entry condition of procedure *transfer* must imply the loop invariant; when the loop terminates the loop invariant and the negation of the loop test ($\neg true$) must imply the exit condition of *transfer*. *outj_0*, *inj_0* are new variables denoting the final values of *outj* and in *j* on exit from the loop.

$$\begin{aligned} & (inj' = inf_0 \wedge \\ & empty(out\ j) \\ & \rightarrow \\ & append(outf, inj) = inf_0 \wedge \\ & (append(outf_0, inj_0) = inf_0 \wedge \\ & \neg true \\ & \rightarrow \\ & outf_0 = inf_0)) \end{aligned}$$

Propositional reasoning reduces this condition to

$$(inf = inf_0 \wedge empty(outf) \rightarrow append(outf, inf) = inf_0)$$

which can be seen to be true by properties of `append`.

4) **Correctness of loop**: The loop invariant must be preserved by executing the loop body:

$$\begin{aligned} & (\mathbf{append(out\ j, in\ j) = in\ f_0} \wedge \\ & \neg \mathbf{empty(in\ j)} \wedge \\ & \mathbf{inj = append(list(c_0), in\ f_{-1})} \wedge \\ & \mathbf{out\ f_{-1} = append(out\ j, list(c_0))} \\ & \rightarrow \\ & \mathbf{append(out\ f_{-1}, in\ f_{-1}) = info}) \end{aligned}$$

After some substitutions this becomes

$$\begin{aligned} & \mathbf{append(out\ j, append(list(c_0), in\ j_{-1})) = in\ f_0} \\ & \rightarrow \\ & \mathbf{append(append(out\ j, list(c_0)), in\ j_{-1}) = in\ f_0} \end{aligned}$$

which is true by associativity of **`append`**.